

# A Mean-field Approach for an Inter-carrier Interference Canceller for OFDM

Ayaka Sakata\* and Yoshiyuki Kabashima

*Department of Computational Intelligence and Systems Science,  
Tokyo Institute of Technology, Midori-ku, Yokohama 226-8502, Japan.*

Yitzhak Peleg

*Department of Physics, Bar-Ilan University,  
Ramat-Gan 52900, Israel.*

(Dated: March 4, 2013)

The similarity of the mathematical description of random-field spin systems to orthogonal frequency-division multiplexing (OFDM) scheme for wireless communication is exploited in an inter-carrier-interference (ICI) canceller used in the demodulation of OFDM. The translational symmetry in the Fourier domain generically concentrates the major contribution of ICI from each sub-carrier in the sub-carrier's neighborhood. This observation in conjunction with mean field approach leads to a development of an ICI canceller whose necessary cost of computation scales linearly with respect to the number of subcarriers. It is also shown that the dynamics of the mean-field canceller are well captured by a discrete map of a single macroscopic variable, without taking the spatial and time correlations of estimated variables into account.

PACS numbers: 05.40.-a, 75.10.Hk, 84.40.Ua, 88.80.ht

## I. INTRODUCTION

Wireless communication technologies play a significant role in the modern information society. As of the end of 2010, there are more than 4.6 billion mobile-cellular subscriptions in the world [1], and the use of wireless devices (such as personal digital assistants and GPS units) is ever increasing. To keep up with the accompanying rapid growth in data traffic, communication efficiency of today's wireless communication systems must thus be constantly improved.

A decade has passed since a fruitful connection between wireless communications and statistical mechanics was introduced by a seminal work by Tanaka [2]. On the basis of an analogy between the demodulation problem of wireless communications and statistical mechanics of disordered Ising spin systems, he successfully clarified the potential efficiency of a wireless communication scheme known as code-division multiple access (CDMA), which is employed in the third generation cellular phone systems. Later, this analogy was also utilized in developing practically feasible and efficient demodulation algorithms for CDMA [3, 4].

In a recent study [5], the connection to statistical mechanics was extended to another wireless-communication scheme, namely, orthogonal frequency-division multiplexing (OFDM), which is today employed in the fourth generation cellular phones and the latest Wi-Fi systems. According to this scheme, the available frequency domain is divided into sub-domains, and data is transmitted by the subcarriers associated with those sub-domains [6]. Because of the orthogonality between the subcarriers, they can be closely placed in the frequency domain,

thereby attaining high-rate-data transmission. However, in a mobile radio environment, relative movement brings about a Doppler spread, which destroys the orthogonality between the subcarriers. This destruction leads to the occurrence of inter-carrier interference (ICI), which rapidly deteriorates the bit error rate. An efficient ICI-cancellation scheme is, therefore, indispensable in use of the OFDM scheme in such environments. In [5], a Monte-Carlo-based ICI cancellation scheme was developed on the basis of mapping an OFDM model to a variant of random-field Ising spin systems. Numerical experiments indicated that the developed scheme can achieve significantly better performance than existing standard methods in terms of bit error rate. However, the cost of computation, which grows with the square of the number of subcarriers, and the technical difficulty in implementing electrical circuits prevent the scheme from being practically significant.

The purpose of this study is to develop an approximate ICI-cancellation scheme for resolving the above-mentioned drawbacks. For this purpose, a mean-field approximation (MFA) is utilized in conjunction with the analogy between OFDM and random field Ising spin systems. Naive MFA requires as much computational cost as the Monte-Carlo based cancellation scheme. We show that utility of translational invariance in the Fourier domain, which is intrinsic in OFDM, makes it possible to develop an ICI cancellation scheme whose computational cost is proportional to the number of subcarriers. We also show that the performance of the developed algorithm based on MFA is well captured by a discrete map of a single variable and that the fact is supported by numerical experiments.

This paper is organized as follows. In section II, we explain the OFDM model studied in this paper. In section III, we propose a MFA-based ICI cancellation scheme. In

---

\* ayaka@sp.dis.titech.ac.jp

section IV, we explore the performance of the proposed method and its time evolution, and derive the approximated expression of the proposed canceller. The derivation of the approximated expression is explained in Appendix A. Finally, section V is devoted to the conclusion and summary.

## II. MODEL

When a time sequence of a signal,  $\mathbf{x} = \{x_t\}$  ( $t = 1, \dots, N$ ), is transmitted in a mobile radio environment, the received symbol,  $\mathbf{y} = \{y_t\}$ , in the multipath channel is expressed by

$$y_t = \frac{1}{\sqrt{M}} \sum_{p=1}^M h_p \exp\left(\sqrt{-1} \frac{2\pi\epsilon_p t}{N}\right) x_t + \eta_t. \quad (1)$$

The time delay of each path is assumed to be zero for simplicity, and the channel noise,  $\{\eta_t\}$ , is independent with respect to the time domain. The number of paths is  $M$ , the amplitude of each path,  $\{h_p\}$ , is distributed according to the Rayleigh distribution [7, 8] as

$$P(h_p) = h_p \exp\left(-\frac{h_p^2}{2}\right), \quad (2)$$

and  $\{\epsilon_p\}$  is the Doppler shift, which is assumed to be distributed uniformly in the region  $\epsilon_p \in [0, \epsilon_{\max}]$ . With this model, the difference between the maximum and minimum values of the Doppler shift is significant irrespective of the sign of the Doppler shift.

By applying the discrete Fourier transformation to eq.(1), the frequency-domain representation of the transmitted signal,  $\mathbf{X} = \{X_k\}$ , and the received signal,  $\mathbf{Y} = \{Y_k\}$ , where  $k = 0, \dots, N-1$  and  $N$  is the number of subcarriers, is given by

$$Y_k = \sum_{l=1}^N W_{kl} X_l + n_k, \quad (3)$$

where  $n_k$  is the discrete Fourier transform of the channel noise. We assume that  $n_k$  is characterized as an additive white Gaussian noise (AWGN) of mean zero and variance  $\sigma_0^2$ . The component of  $N \times N$  matrix  $\mathbf{W}$ , called a frequency-domain matrix, is given by

$$W_{kl} = \sum_{p=1}^M \frac{h_p \sin(\pi(l-k+\epsilon_p)) e^{\sqrt{-1}(1-\frac{1}{N})\pi(l-k+\epsilon_p)}}{\sqrt{MN} \sin(\pi(l-k+\epsilon_p)/N)}, \quad (4)$$

where  $W_{kl}$  represents the intensity of the interference from subcarrier  $l$  to  $k$ . When  $\{\epsilon_p\} = \mathbf{0}$ , the matrix is diagonal, namely  $W_{kl} = \delta_{k,l}$ , where  $\delta$  is Kronecker's delta, and there is no ICI between any subcarriers. In general, the frequency-domain matrix has translation symmetry, so the value of each component  $W_{kl}$  only depends on the

difference of the indices, namely  $k-l$ . This fact is a result of the Fourier representation.

The Doppler shift,  $\epsilon_p$ , is normalized by the frequency separation of subcarriers,  $\Delta f = 1/N$ , as  $\epsilon_p = f_{D,p}/\Delta f$ , where  $f_{D,p}$  is the Doppler frequency at  $p$ -th path. The parameter  $\epsilon_p$  indicates the influence of the Doppler effect on ICI with a given alignment of subcarriers.

The transmitted bits, channel noise, and received bits are represented as complex numbers. Their real and imaginary parts are denoted by  $\mathbf{X}^R$  and  $\mathbf{X}^I$ ,  $\mathbf{n}^R$  and  $\mathbf{n}^I$ , and  $\mathbf{Y}^R$  and  $\mathbf{Y}^I$ , respectively. Hereafter, they are represented as vectors consisting of  $2N$  elements:  $\mathbf{X} \equiv [\mathbf{X}^R, \mathbf{X}^I]^T$ ,  $\mathbf{n} \equiv [\mathbf{n}^R, \mathbf{n}^I]^T$ , and  $\mathbf{Y} \equiv [\mathbf{Y}^R, \mathbf{Y}^I]^T$ , respectively.  $T$  denotes the operation of the matrix transpose. The corresponding frequency-domain matrix is redefined as a  $2N \times 2N$  matrix,  $\mathbf{W} \equiv \begin{bmatrix} \mathbf{W}^R & -\mathbf{W}^I \\ \mathbf{W}^I & \mathbf{W}^R \end{bmatrix}$ , where  $\mathbf{W}^R$  and  $\mathbf{W}^I$  are the real and imaginary parts of the matrix, respectively [5].

## III. MEAN-FIELD ICI CANCELLER

The problem of the OFDM system is to recover the original signal by canceling out the inter-carrier interference. The Bayesian framework offers various ICI canceling strategies on the basis of the posterior distribution,

$$P(\mathbf{X}|\mathbf{Y}) = \frac{P(\mathbf{Y}|\mathbf{X})P(\mathbf{X})}{\sum_{\mathbf{X}} P(\mathbf{Y}|\mathbf{X})P(\mathbf{X})}, \quad (5)$$

where the likelihood  $P(\mathbf{Y}|\mathbf{X})$  is given by the distribution of the channel noise [9]. At the receiving side, it is assumed that the channel noise is described by the Gauss distribution with mean zero and variance  $\sigma^2$ ,

$$P(\mathbf{Y}|\mathbf{X}) = \left(\frac{1}{2\pi\sigma^2}\right)^{N/2} \exp\left(-\frac{(\mathbf{W}\mathbf{X} - \mathbf{Y})^2}{2\sigma^2}\right), \quad (6)$$

and prior distribution  $P(\mathbf{X})$  is the uniform distribution. The posterior probability can therefore be expressed as

$$\begin{aligned} P(\mathbf{X}|\mathbf{Y}) &= \frac{1}{Z} \exp\left\{\frac{1}{\sigma^2} \left(\frac{1}{2} \sum_{i,j} J_{ij} X_i X_j - \sum_{i=1}^{2N} h_i X_i\right)\right\} \\ &\equiv \frac{1}{Z} e^{-\beta \mathcal{H}(\mathbf{X}|\mathbf{J}, \mathbf{h})} \end{aligned} \quad (7)$$

where  $\sigma^{-2}$  is identified with the ‘‘inverse temperature’’  $\beta$ , the interaction matrix and the external field are given by  $\mathbf{J} = \mathbf{W}^T \mathbf{W}$  and  $\mathbf{h} = \mathbf{Y}^T \mathbf{W}$ , respectively, and  $Z$  corresponds to the partition function. The interaction matrix,  $\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 \\ \mathbf{J}_2^T & \mathbf{J}_1 \end{bmatrix}$ , also has translational invariance; the  $(k, l)$ -components of  $N \times N$  sub-matrices  $\mathbf{J}_1$  and  $\mathbf{J}_2$  only depend on  $k-l$ .

The Hamiltonian of the OFDM system,  $\mathcal{H}(\mathbf{X}|\mathbf{J}, \mathbf{h})$ , defined in eq.(7), can be regarded as that for a random-field spin model. Unlike typical random field models, the

random field of the OFDM model is determined by the ICI between transmitted bits and the channel's properties. In particular, when the number of the paths,  $M$ , is equal to 1, the value of the off-diagonal element is much smaller than that of the diagonal element; hence, the system can be regarded as a single-body problem with random fields.

The maximum a posterior probability (MAP) strategy

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X}} P(\mathbf{X}|\mathbf{Y}), \quad (8)$$

where  $\arg \max$  is the argument giving the maximum value of the function, is guaranteed to minimize the block-wise error probability. This strategy corresponds to the search of the ground state of Hamiltonian  $\mathcal{H}(\mathbf{X}|\mathbf{J}, \mathbf{h})$ . However, the numerical cost of exactly obtaining the maximizer  $\hat{\mathbf{X}}$  increases exponentially with increasing number of subcarriers. Zero temperature ( $\beta \rightarrow \infty$ ) synchronous dynamics of  $N$  symbols based on the mean-field approximation (MFA) is a practically feasible approximate scheme for finding the MAP solution of eq.(8) [10]. In the case of the quadrature-phase-shift-keying (QPSK) modulation, where the components of  $\mathbf{X}$  takes one of the two values,  $\pm 1$ , the scheme is expressed as

$$\hat{X}_k^{t+1} = \text{sgn}(h_k - \sum_{l \neq k} J_{kl} \hat{X}_l^t), \quad (9)$$

where  $\text{sgn}(u)$  denotes the sign of  $u$  and  $\hat{X}_k^t$  is the tentative decision of the  $k$ -th symbol after  $t$  iterations. It is assumed that the configuration  $\{\hat{X}_k^t\}$  is invariant at  $\{\hat{X}_k^*\}$  after sufficient updates, and the fixed configuration is regarded as the final decision of the transmitted bits. Synchronous update schemes similar to eq.(9) have been introduced for evaluating the minimum mean square error (MMSE) estimator of Gaussian priors  $P(\mathbf{X}) \propto \exp(-|\mathbf{X}|^2/(2\sigma_X^2))$ , in which the transmitted bits are estimated as  $\hat{\mathbf{X}}_{\text{MMSE}} = ((\sigma^2/\sigma_X^2)\mathbf{I} + \mathbf{J})^{-1} \mathbf{h}$  in the current case, where  $\mathbf{I}$  is the identity matrix [11–13].

The computational cost of eq.(9) increases as  $O(N^2)$ , and it may reduce the practical feasibility of eq.(9). To reduce this cost increase, the proposed algorithm utilizes the fact that the absolute value of  $J_{1kl}$  and  $J_{2kl}$  decreases as  $|k-l|$  increases, which indicates that the major contributions to ICI (to which each subcarrier is subject) are concentrated on the subcarrier's neighborhood. Based on this observation, the strategy proposed here considers only a part of the ICI among subcarriers in the frequency domain at each stage of the cancellation. A similar strategy was also proposed for a Gaussian MMSE estimator [14].

Let us define the set of indices of subcarriers that are considered to be contributed on ICI of subcarrier  $k$  as  $\partial_k(\omega) \equiv \{x, x+N | x = \text{mod}(k+N-1 \pm y, N) + 1, y \in \mathbb{N}, 1 \leq y \leq \omega\}$ , where  $\text{mod}(k, N)$  means the remainder of  $k/N$  as an integer. In general,  $\partial_k(\omega)$  is composed of  $4\omega$  elements; for instance,  $\partial_1(1) = \{2, N, N+2, 2N\}$ . The

update rule is then given as

$$\hat{X}_k^{t+1} = \text{sgn}(h_k - \sum_{l \in \partial_k(\omega)} J_{kl} \hat{X}_l^t). \quad (10)$$

The algorithm (9) corresponds to the case that  $\omega = \lfloor N/2 \rfloor$  in eq.(10), where  $\lfloor x \rfloor$  denotes the largest integer not greater than real number  $x$ . The numerical cost per iteration of eq.(10) is  $O(N)$  as long as parameter  $\omega$  is  $O(1)$ . When  $\omega < N/2$ , fixed point  $\{\hat{X}_i^*\}$  does not correspond to the maximizer of the posterior probability, but it is expected to be a good approximation of the maximizer.

## IV. RESULTS

### A. Performance of ICI canceller

We observe bit error rate (BER), which is defined by

$$\text{BER} = \frac{1}{2N} \sum_{i=1}^{2N} \langle \hat{X}_i^* X_i \rangle, \quad (11)$$

where  $\overline{\cdots}$  and  $\langle \cdots \rangle$  represent the average over the frequency-domain matrix  $\mathbf{W}$  and over the channel noise and transmitted symbol, respectively. The BER performance of the decoder is bounded from below by that for a single bit transmitted through the AWGN channel, which is given by

$$\text{BER}^{\text{OPT}} = \frac{1}{2} \text{erfc}(\sqrt{\text{Eb}/\text{N0}}), \quad (12)$$

because there is no ICI involved when only a single bit is transmitted. BER of the ICI canceller closes to  $\text{BER}^{\text{OPT}}$  as the elimination of ICI becomes successful. In the current model, the signal to noise ratio (SNR) is given by,

$$\text{SNR} = \frac{1}{N} \sum_{i,j=1}^N \frac{|W_{ij}|^2}{\sigma_0^2}. \quad (13)$$

In the QPSK modulation case, the number of bits per symbol is two, so  $\text{Eb}/\text{N0}$  corresponds to  $\text{SNR}/2$  [7].

We check the performance of the matched filter, whose mathematical manipulation corresponds to multiply the received symbol by the Hermit conjugate matrix of  $\mathbf{W}$ . The matched filter minimizes the power of the channel noise, but it cannot reduce the error due to the intercarrier interference. Therefore, as a canceller of intercarrier interference, the performance of the proposed algorithm should be better than that of the matched filter. In the proposed algorithm,  $\omega = 0$  corresponds to the demodulation by the matched filter and as  $\omega$  increases, the performance is expected to improve.

$\text{Eb}/\text{N0}$  dependence of BER is shown in Fig.1 for numbers of the path, (a)  $M = 3$  and (b)  $M = 15$ , respectively. The number of the subcarriers is  $N = 32$ , and the maximum value of the Doppler shift is  $\epsilon_{\text{max}} = 0.5$ .

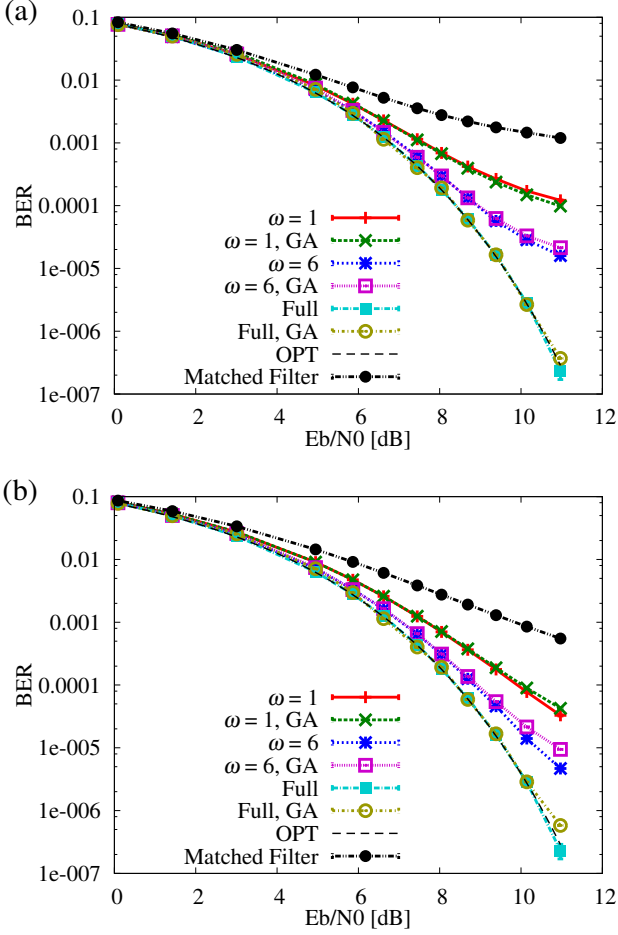


FIG. 1. (color online) Dependence of BER on  $E_b/N_0$  at (a)  $N = 32$ ,  $M = 3$ ,  $\epsilon_{\max} = 0.5$  and (b)  $N = 32$ ,  $M = 15$ ,  $\epsilon_{\max} = 0.5$ . Results for  $\omega = 1$  and  $\omega = 6$  are shown and “Full” corresponds to  $\omega = 16$ . The dashed line and dashed-dotted line represent the optimal and matched filter cases, respectively. GA means the BER performance under the Gaussian approximation. The data points are averaged over  $10^6$  samples of  $\mathbf{W}$ .

$\text{BER}^{\text{OPT}}$  and BER for the matched filter are shown by the dashed line and dashed-dotted line, respectively. The BER performance of the canceller given by eq.(10) with  $\omega = 1$ , which is the simplest case, is better than that of the matched filter, and it improves as the number of interactions increases. When all interactions between the subcarriers are taken into account, the BER performance of the canceller given by eq.(9) almost coincides with the optimal performance  $\text{BER}^{\text{OPT}}$ .

The  $\omega$ -dependence of BER performance at  $M = 3$ ,  $\epsilon_{\max} = 0.5$ , and  $E_b/N_0 = 8.69$  is shown in Fig.2 for  $N = 64, 128$ , and  $256$ . This graph indicates that BER does not depend on the number of subcarriers,  $N$ . BER rapidly decreases as  $\omega$  increases from zero, and it gradually approaches  $\text{BER}^{\text{OPT}}$  as  $\omega$  further increases. At  $\omega \gtrsim 16$ , the differences between the BER of the proposed

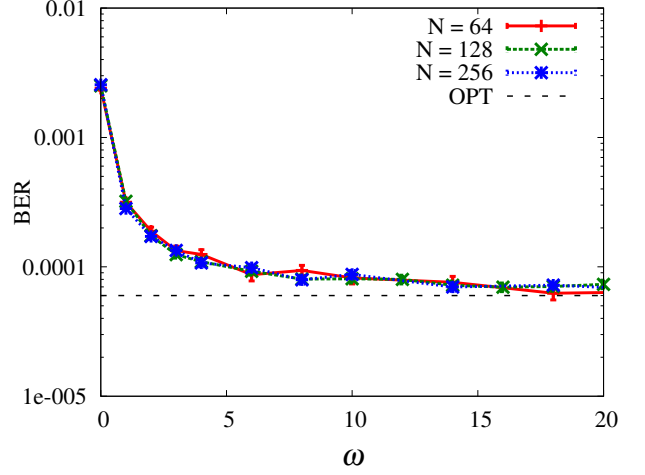


FIG. 2. (color online) Dependence of the BER performance on  $\omega$  at  $M = 3$ ,  $\epsilon_{\max} = 0.5$ , and  $E_b/N_0 = 8.69$  dB. Results for numbers of subcarriers  $N = 64, 128$ , and  $256$  are shown, and the dashed line shows the BER of the optimal limit.

method and the optimal limit is less than 10% of the value of  $\text{BER}^{\text{OPT}}$ . This result indicates that  $\omega \sim 16$  is sufficient to practically achieve the BER of the original MFA based canceller (9) irrespective of  $N$ ; thus, the required numerical cost per bit does not increase as  $O(N)$ . In this model, the unit width of the frequency domain is given by  $1/N$ , then the proposed algorithm will provide more effective use of the frequency domain without increasing the numerical cost.

## B. ICI dynamics

To determine the validity of the proposed method as a realistic canceller, the time evolution of the BER of the proposed algorithm, decoding algorithm at  $N = 128$ ,  $M = 15$ , and  $\epsilon_{\max} = 0.5$  is plotted in Fig.3. The initial condition,  $\{\hat{X}_k^0\}$ , is obtained by the matched filter, so BER at the 0-th step corresponds to that at the fixed point for  $\omega = 0$ . The horizontal axis means the number of time steps, and the vertical axis represents the BER with respect to the tentative decision at the time step. As can be seen in the figure, the BER performances at  $\omega = 3$  and  $\omega = 64$  (full) converge to equilibrium values after two updates. The time steps required to reach the fixed point are quite short and only slightly depend on  $N$ , and the proposed algorithm is useful for an implementation as an ICI canceller.

## C. Macroscopic description

To analyze the time evolution of the canceller given by eq.(10), we attempt to describe the dynamics by using

a finite number of macroscopic variables [4, 15]. The overlap between the transmitted bit and the predicted bit at step  $t$  under a given realization of  $\mathbf{W}$  is defined as

$$m^t = \frac{1}{2N} \sum_{k=1}^{2N} \langle \hat{X}_k^t X_k \rangle, \quad (14)$$

and the BER at step  $t$  is given by  $0.5 \times (1 - m^t)$ . The simplest description of the time evolution of eq.(14) is provided by ignoring all spatial/time correlations among the subcarrier symbols, which leads to a discrete map of  $m^t$ :

$$m^{t+1} = \int Dz \operatorname{sgn}(z_0 + \sqrt{\Sigma_t^2} z), \quad (15)$$

$$z_0 = \frac{1}{2N} \sum_{k=1}^{2N} J_{kk}, \quad \Sigma_t^2 = 2A(1 - m^t) + B + \sigma^2 C,$$

where  $Dz = dz e^{-z^2/2} / \sqrt{2\pi}$ , and the coefficients for the fixed sample of  $\mathbf{W}$  are given by

$$A = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{l \in \partial_k(w)} J_{kl}^2, \quad (16)$$

$$B = \frac{1}{2N} \sum_{k=1}^{2N} \sum_{l \notin \partial_k(w)} J_{kl}^2, \quad (17)$$

$$C = \frac{1}{2N} \sum_{k=1}^{2N} \sum_l W_{lk}^2 = z_0. \quad (18)$$

The derivation is given in detail in Appendix A. These expression indicate that the ICI is approximated by Gaussian noise, and noise variance  $\sigma^2$  is effectively increased by  $B/C$  due to the insufficiency of the ICI cancellation of the MFA canceller (10). The macroscopic equation corresponding to eq.(9) is obtained by setting  $\omega = N/2$ .

BER defined at fixed points given by eq.(15), denoted by GA (Gaussian approximation), are compared to the real BER curve in Fig.1, where  $\text{BER} = (1 - m_t)/2$ . BER at the fixed point are in good accordance with the experimental data irrespective of the value of  $\omega$ . The time evolution of BER is also well described by eq.(15), as shown in Fig.3, in which the initial condition,  $\{m_i^0\}$ , is chosen to correspond to the BER of the matched filter. These results differ substantially from that of the random spreading codes, where time correlation plays a significant role in the macroscopic dynamics [4]. The difference implies that the orthogonality between the subcarriers in OFDM reduces the time correlation and enables the ICI dynamics to converge within a few steps.

## V. CONCLUSION

A practically feasible ICI canceller for the OFDM model, which can be regarded as a variant of the random

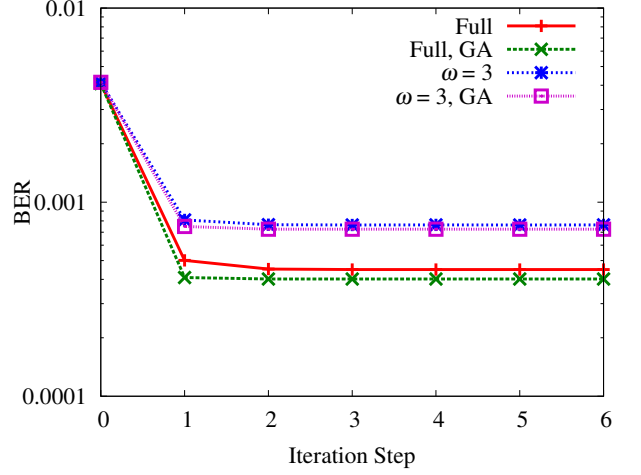


FIG. 3. (color online) The time-evolution of BER at  $N = 128$ ,  $M = 15$ ,  $\epsilon_{\max} = 0.5$ , and  $E_b/N_0 = 7.45\text{dB}$ . Results for  $\omega = 3$  and full case ( $\omega = 64$ ) are shown, and GA means the corresponding Gaussian approximation. The trajectory is averaged over  $10^4$  samples of  $\mathbf{W}$ .

field spin model, was developed. The cancellation scheme was derived by applying the mean-field approximation to the maximization of the posterior probability which corresponds to the search of the ground state of the spin model. The properties of the frequency-domain matrix, i.e., translation symmetry and smallness of off-diagonal elements compared to diagonal elements, was focused on, and only a part of the ICI among the subcarriers located within a distance  $\omega$  in the frequency domain for each bit was considered. The numerical cost of the ICI canceller is thus controlled by the parameter  $\omega$ . When  $\omega = \lfloor N/2 \rfloor$ , the ICI between all subcarriers is considered, and the ICI canceller corresponds to the approximated MAP demodulator.

The BER of the proposed algorithm used for QPSK modulation is better than the matched filter even if  $\omega = 1$ , and it practically approaches the optimal limit as  $\omega$  increases further. The BER performance is saturated near the optimal limit at around a certain  $\omega \sim O(1)$  in the whole  $E_b/N_0$  region. Furthermore, the performance under a given  $\omega$  only slightly depends on  $N$ . This result means that the required numerical cost per iteration to achieve a feasible performance level is  $O(N)$ .

The number of time steps required to reach a fixed point of ICI canceller is  $O(1)$  only slightly depends on the value of  $N$  and  $\omega$ . The total numerical cost to eliminate ICI and demodulate the transmitted bits is therefore  $O(N)$ . The proposed algorithm will be practical to implement by virtue of its low computational cost.

The fixed point of the ICI canceller and the dynamics to reach there are well described by a discrete map of a single macroscopic variable under the approximation of ICI for each bit as independent Gaussian noise. It is considered that the orthogonality between the subcar-



riers prevents a time correlation being induced, and is a mathematical background of the high accuracy of the proposed ICI canceller.

The proposed algorithm is efficient for the parameter region where the Doppler shift causes large intercarrier interference. By introducing this algorithm as an ICI canceller, the OFDM scheme is useful when subcarriers are closely arranged in the frequency domain and a mobile object moves at high speed. More efficient use of a given frequency domain and enhanced accuracy in satellite communications are also expected.

## ACKNOWLEDGMENTS

We would like to thank Ido Kanter for his helpful comments and discussions. This work was supported by JSPS Fellow No. 23-4665 (AS) and KAKENHI No. 22300003 (YK).

## Appendix A: Derivation of macroscopic dynamics

According to the definition of  $\{h_k\}$ , eq.(10) can be transformed as follows,

$$\hat{X}_i^{t+1} = \text{sgn}[-\sum_{j \in \partial_i(\omega)} J_{ij} \hat{X}_j^t + \sum_{j=1}^{2N} J_{ij} X_j + \sum_{\mu=1}^{2N} W_{\mu i} n_\mu]. \quad (\text{A1})$$

By separating the summation of the first and second term into three part;  $j \in \partial_i(\omega)$ ,  $j \notin \partial_i(\omega)$  and  $j = i$ , the time evolution of  $m_i^t$  can be written as

$$\begin{aligned} m_i^{t+1} &= \langle \text{sgn}[J_{ii} + X_i \sum_{j \in \partial_i(\omega)} J_{ij} (X_j - \hat{X}_j^t) \\ &\quad + X_i \sum_{j \notin \partial_i(\omega)} J_{ij} X_j + X_i \sum_{\mu} W_{\mu i} n_\mu] \rangle \\ &\equiv \langle \text{sgn}[J_{ii} + X_i z_i^t] \rangle. \end{aligned} \quad (\text{A2})$$

The right-hand side of eq.(A2) depends on the randomness; the transmitted symbol and the channel noise, through  $z_i^t$ . The average over the randomness can therefore be replaced by the average over  $z_i^t$  according to an appropriate distribution. The distribution of  $z_i^t$  is approximated by a Gauss distribution. The first and second moments of  $z_i^t$  are given by

$$\langle z_i^t \rangle = 0, \quad (\text{A3})$$

$$\langle z_i^{t2} \rangle = 2 \sum_{j \in \partial_i(\omega)} J_{ij}^2 (1 - m_j^t) + 2 \sum_{j \notin \partial_i(\omega)} J_{ij}^2 + \sigma^2 \sum_{\mu=1}^{2N} W_{\mu i}^2, \quad (\text{A4})$$

where it is assumed that  $X_j = \hat{X}_j^t$  with probability  $(1 + m_j^t)/2$ , and  $X_j = -\hat{X}_j^t$  with probability  $(1 - m_j^t)/2$  for

any  $j$ . With these quantities, the following expression can be obtained:

$$m_i^t = \int Dz \text{sgn}[J_{ii} + X_i \sqrt{\Sigma_i^{t2}} z], \quad (\text{A5})$$

where  $\Sigma_i^{t2} = \langle z_i^{t2} \rangle - \langle z_i^t \rangle^2$ , and coefficient  $X_i$  can be ignored because the function is invariant against translation  $z \rightarrow -z$ . The approximated expression for the full case can be obtained by setting  $\omega = N/2$ , so  $\partial_i(\omega)$  contains all bits except  $i$ .

- 
- [1] *Robust demand for mobile phone service will continue*, UN News Centre, February **15**, (2010).
  - [2] T. Tanaka, Europhys. Lett. **54**, 540 (2001).
  - [3] Y. Kabashima, J. Phys. A: Math. Gen. **36**, 11111 (2003).
  - [4] T. Tanaka, and M. Okada, IEEE Transactions on Information Theory, **51**, 700 (2005).
  - [5] H. Efraim, Y. Peleg, I. Kanter, O. Shental, and Y. Kabashima, Phys. Rev. E **82**, 060101 (2010).
  - [6] R. W. Chang, and R. A. Gibby, IEEE Trans. Commun. **COM-16**, 529 (1968).
  - [7] Y. Zhao, and S.-G. Häggman, IEEE Trans. Veh. Technol. **3**, 1564 (1996).
  - [8] Y. Zhao, and S.-G. Häggman, IEEE Trans. Commun. **49**, 1185 (2001).
  - [9] J. Li, K. B. Letaief, R. S. Cheng, and, Z. CAO, IEEE Veh. Tech. **3**, 1553 (2001).
  - [10] M. K. Varanasi, and B. Aazhang, IEEE Trans. Commun., **38**, 509 (1990).
  - [11] A. Gorokhov, and J.-P. Linnartz, IEEE Trans. Commun. **52**, 572 (2004).
  - [12] W.-S. Hou, and B.-S. Chen, IEEE Trans. Wire. Commun. **4**, 2100 (2005).
  - [13] A. F. Molisch, M. Toeltsch, and S. Vermani, IEEE Trans. Veh. Tech. **56**, 2158 (2007).
  - [14] P. Schniter, IEEE Trans. Sign. Proc., **52**, 1002 (2004).
  - [15] M. Okada, Neural Networks **8**, 833 (1995).